Close Thu: HW 13.2   Close Fri: HW 13.3

Exam 2 is Tues. It covers

10.1-10.3, 12.4: Analyzing functions

11.1/2: Derivatives involving e^x and ln(x)

12.1,12.3: Antiderivatives, finding C

13.2-13.3: Definite Integrals and areas

13.2 Definite Integrals (Continued)

Entry Task: Evaluate

1. $\int_1^5 \frac{3}{4x^2}\, dx$

2. $\int_0^1 e^{x/3}\, dx$

Recall:

Fundamental Theorem of Calculus

If $F(x)$ is *any* anti-derivative of $f(x)$,then

$$\int_a^b f(t)dt = F(b) - F(a)$$

Step 1: Find *any* antiderivative, $F(x)$.

Step 2: Compute $F(b)$ and $F(a)$

Step 3: Subtract

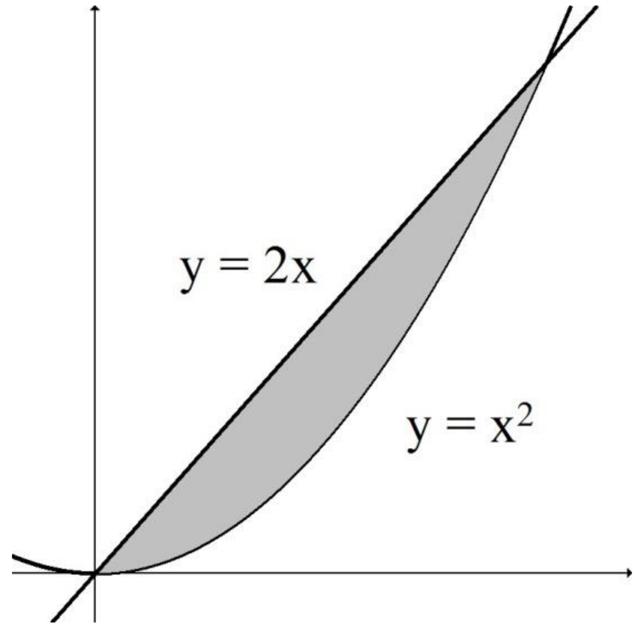
$$\int_a^b f(x)dx = \quad F(x)\Big|_a^b = F(b) - F(a)$$

3. $\int_{1}^{4} \sqrt{x}\, dx$

4. $\int_{1}^{e} \frac{5}{x}\, dx$

13.3 Area Between Curves and Applications

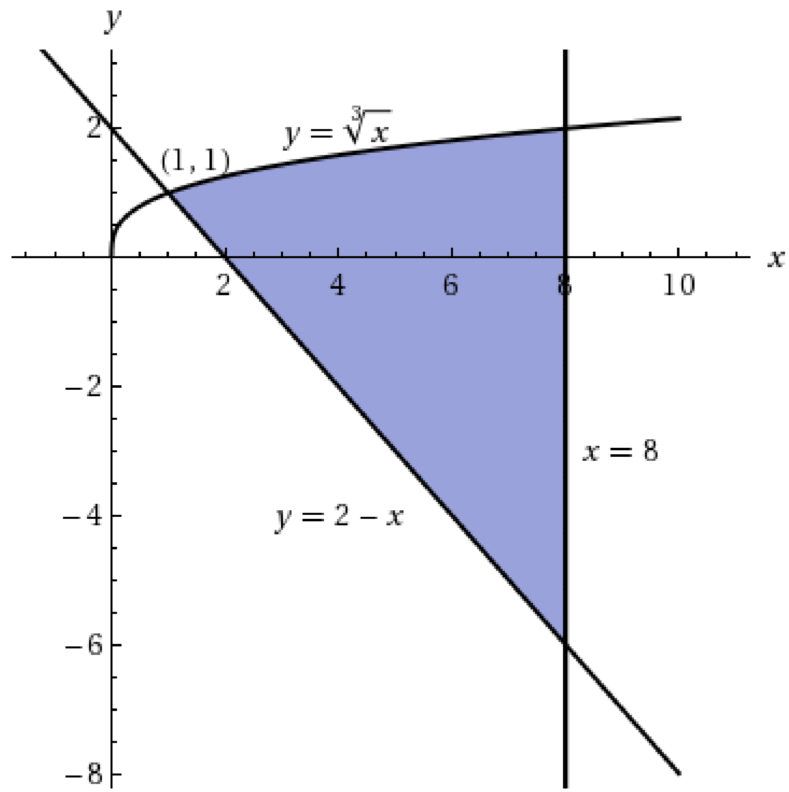
Example: Find the area bounded between $y = 2x$ and $y = x^2$.



Example (from HW):

Find the area of the region bounded by

$$y = \sqrt[3]{x}, y = 2 - x \text{ and } x = 8.$$



<u>To find area between curves</u>

1. Draw an accurate picture.
   Find intersections and identify

   $f(x) =$ "top function"

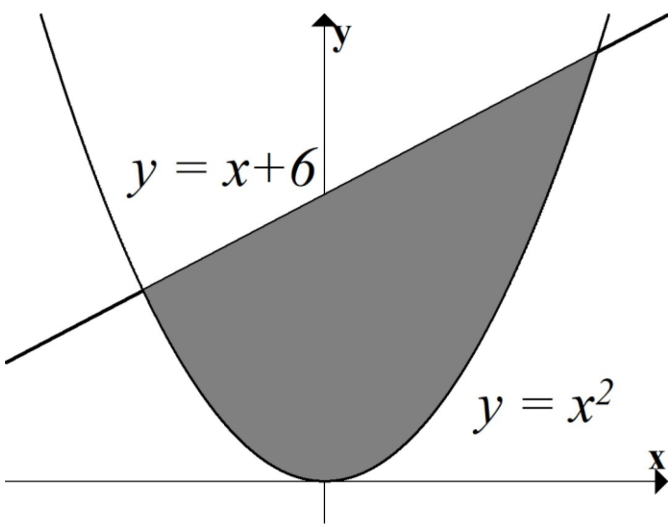
   $g(x) =$ "bottom function"
2. Compute:

$$\int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b f(x) - g(x)dx$$

Old Exam Question:

Find the area of the region bounded by

$$y = x^2 \text{ and } y = x + 6$$

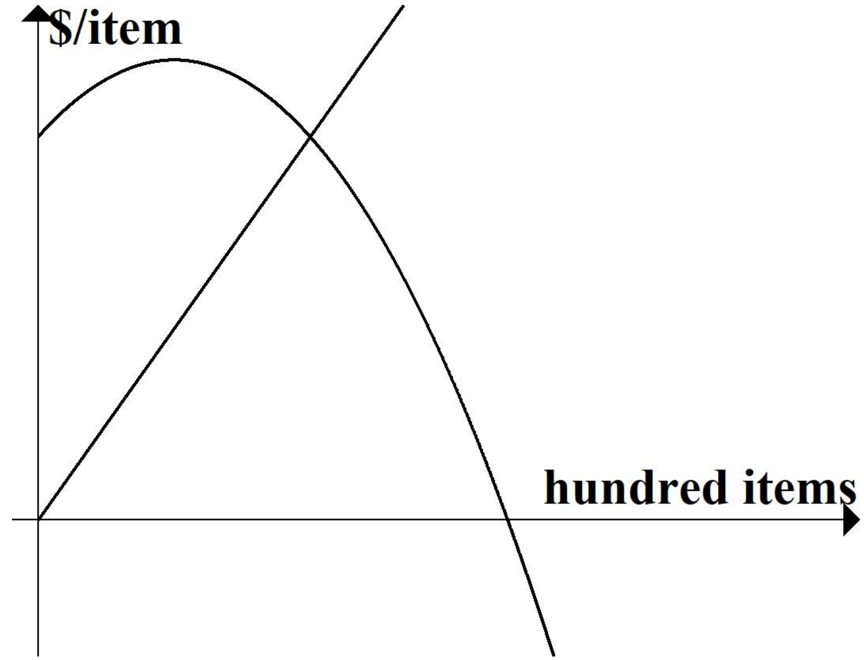


Example: Suppose

$MR(x) = -x^2 + 2x + 5$ dollars/item

$MC(x) = \frac{5}{2}x$ dollars/item

where *x* is in hundreds of items, and assume *FC* = 3 hundred dollars.



What do the following represent?

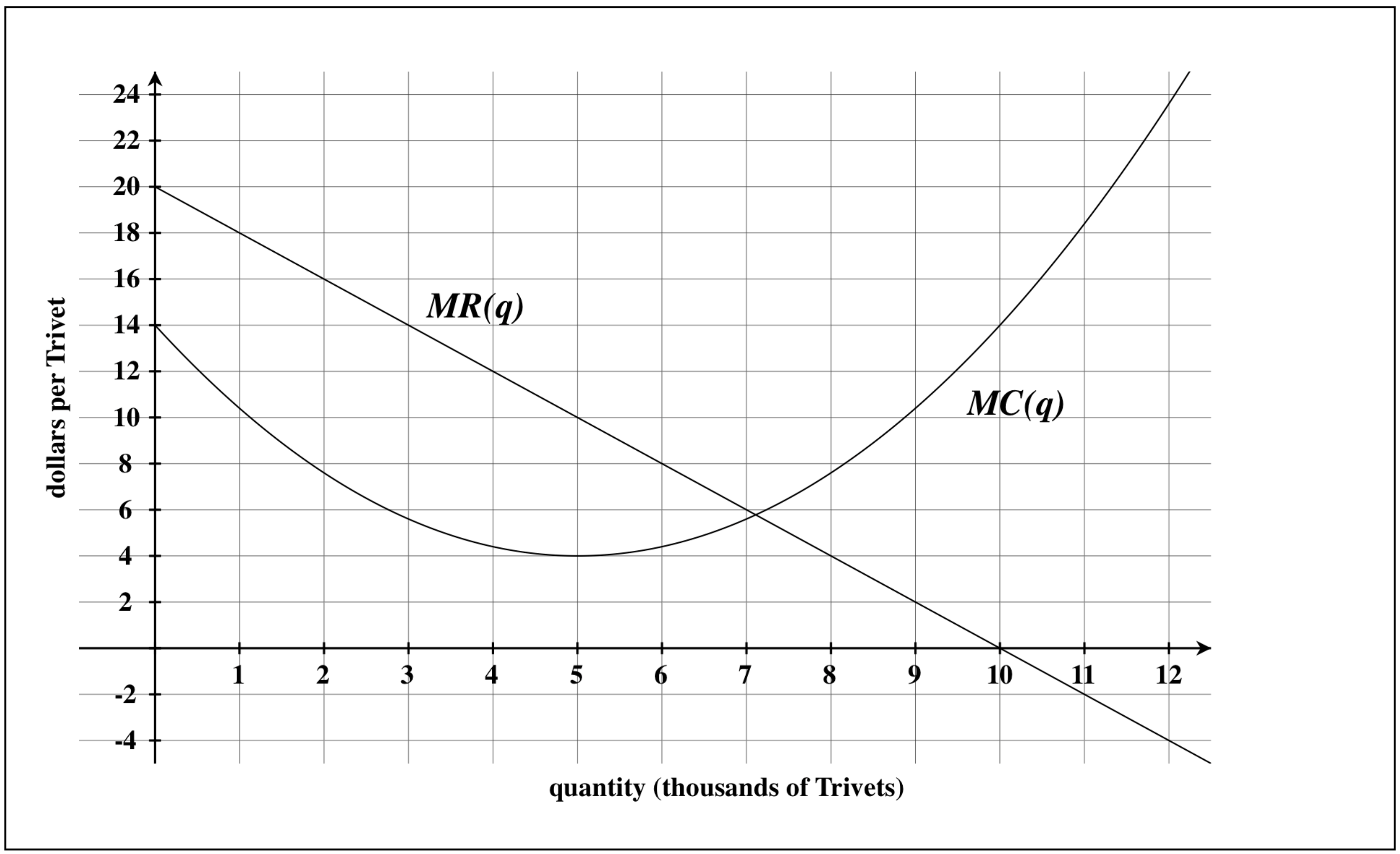
a. Area under MR from 0 to 2.
b. Area under MC from 0 to 2.
c. Area between MR & MC from 0 to 2.

Note:
The area between f(x) and g(x) gives ***change in difference between anti-derivatives*** from $x = a$ to $x = b$.

If you want to get Profit directly from the graph of MR and MC:

1. Find the area between MR and MC from 0 to your desired quantity.
2. If MR > MC treat it as positive.
3. If MR < MC treat it as negative.
4. Don't forget to subtract FC.

Example (from HW):

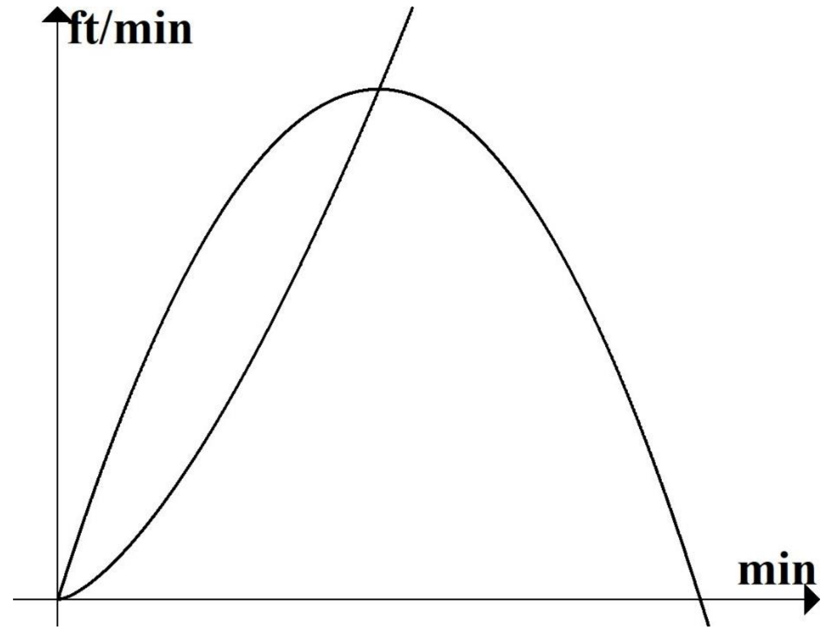


Example:

At time t = 0 minutes, a Red and a Green balloon are next to each other at a height of 60 feet. The ***rate of ascent*** of each balloon is given by

$$R'(t) = -\frac{1}{2}t^2 + 4t \qquad \text{feet/min}$$

$$G'(t) = t^{3/2} \qquad \text{feet/min}$$



These graphs intersect at t = 4 minutes.

What do the following represent?

a. Area under $R'(t)$ from 0 to 4.

b. Area under $G'(t)$ from 0 to 4.

c. Area between from 0 to 4.

Note: The last example is the exact same idea as getting profit from MR and MC.

If you want to get *distance between* two balloons directly from the graphs of their derivatives:

1. Find between the derivatives from 0 to the desired time.
2. Whatever deriv. is on top is the balloon going faster (treat that area as positive if that is the balloon you are treating as ahead).

Example: Find the area of the region bounded between these curves.

$$y = x^2 - 8x + 24$$

$$y = -x^2 + 8x.$$

You do: Find the area of the region bounded by the y-axis and

$$y = 14 - 2x$$
$$y = 2 + x.$$

If *x* is in hundreds of items and

$$y = MR(x) = 14 - 2x \qquad \$/\text{item.}$$
$$y = MC(x) = 2 + x \qquad \$/\text{item.}$$

What does the area you just found represent? What additional information would you like to know?